MA262 — REVIEW PROBLEMS FOR EXAM II

1. Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix} \quad v_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$$

The dimension of the vector space span$\{v_1, v_1, v_3, v_4\}$ is equal to

A. 1

B. 2

C. 3

D. 4

E. 5

2. Find all values of k such that the vectors $v_1 = (1, -1, 0)$ $v_2 = (1, 2, 2)$ and $v_3 = (0, 3, k)$ form a basis for $\mathbb{R}^3$.

A. $k = 1$

B. $k = 2$

C. $k \neq 1$

D. $k \neq 2$

E. $k \neq 3$

3. Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

If the determinant of A is equal to five and the determinant of B is equal to 6 the determinant of the matrix

$$C = \begin{bmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 3b_{11} & b_{12} + 2b_{11} & b_{13} \\ 3b_{21} & b_{22} + 2b_{21} & b_{23} \\ 3b_{31} & b_{32} + 2b_{31} & b_{33} \end{bmatrix}$$

is equal to

A. 180

B. 100

C. 150

D. 200

E. 300

4. If $y(x)$ is the solution of

$$y'' - 2y' + y = 0,$$
$$y(0) = 1, \quad y'(0) = -1$$

then $y(\frac{1}{2})$ is equal to

A. 0

B. $e^{\frac{1}{2}}$

C. $2e^{\frac{1}{2}}$

D. $\dfrac{1}{2}e^{\frac{1}{2}}$

E. $-3e^{\frac{1}{2}}$

7. The constants a, b, c, d are real numbers and the solutions of the polynomial equation $p(r) = ar^3 + br^2 + cr + d = 0$, are given by $r_1 = 1 + 2i$, $r_2 = 1 - 2i$ and $r_3 = 4$. We can say that the general solution of the equation $ay^{(3)} + by^{(2)} + cy' + dy = 0$ is given by

A. $y(x) = C_1 e^{4x} + C_2 e^{x} \cos(2x) + C_3 e^{x} \sin(2x)$

B. $y(x) = C_1 e^{4x} + C_2 e^{2x} \cos(2x) + C_3 e^{2x} \sin(2x)$

C. $y(x) = C_1 e^{4x} + C_2 e^{4x} \cos(x) + C_3 e^{4x} \sin(x)$

D. $y(x) = C_1 e^{4x} + C_2 e^{2x} \cos(x) + C_3 e^{2x} \sin(x)$

E. $y(x) = C_1 e^{4x} + C_2 e^{x} \cos(x) + C_3 \sin(x)$

8. We know that the characteristic polynomial of a certain homogeneous differential equation is given by

$$p(r) = (r^2 - 1)^2 (r^2 + 4)^2.$$

We can say that the general solution of the differential equation is given by

A. $C_1 e^{x} + C_2 x e^{x} + C_3 e^{-x} + C_4 x e^{-x} + C_5 \cos(2x) + C_6 x \cos(2x) + C_7 \sin(2x) + C_8 x \sin(2x)$

B. $C_1 e^{x} + C_2 x e^{x} + C_3 e^{-x} + C_4 x e^{-x} + C_5 (1 + x) \cos(2x) + C_6 (1 + x) \sin(2x)$

C. $C_1 e^{x} + C_2 x e^{x} + C_3 x^2 e^{x} + C_4 (1+x) e^{-x} + C_5 \cos(2x) + C_6 x \cos(2x) + C_7 x^2 \cos(2x) + C_8 x \sin(2x)$

D. $C_1 e^{x} + C_2 e^{-x} + C_3 \cos(2x) + C_4 \sin(2x)$

E. $C_1 e^{x} + C_2 e^{-x} + C_3 \cos(2x) + C_4 x \cos(2x) + C_5 \sin(2x) + C_6 x \sin(2x)$

F. $C_1 e^{x} + C_2 x e^{x} + C_3 e^{-x} + C_4 x e^{-x} + C_5 \cos(2x) + C_6 x \sin(2x) + C_7 x^2 \sin(2x)$

9. The Wronskian of the functions $\{x, \sin x, \cos x\}$ (in this order) is equal to

A. x

B. $-x$

C. $x \sin x + \cos x$

D. $\sin x \cos x$

E. $x(\cos^2 x - \sin^2 x)$

6. The following system of linear equations has an infinite number of solutions:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= 0, \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= 1, \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= 0, \end{aligned}$$

In this case the matrix $A = (a_{ij}) \in R^{3\times 3}$ satisfies the condition

A. A is defective Ignore this option. We will see this later

B. A is nonsingular

C. The homogeneous systems $A\mathbf{x} = 0$ has exactly one solution

D. The homogeneous system $A\mathbf{x} = 0$ has more than one solution

E. None of the above

7. Let A be a 4×4 matrix. If $\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$ are solutions of the system of linear equations $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$, then which of the following is also a solution?

A. $\mathbf{p} - \mathbf{q}$

B. $\mathbf{q} - 2\mathbf{p}$

C. $2\mathbf{p} - \mathbf{q}$

D. All of the above

E. None of the above

8. The entry b_{32} of the matrix $A^{-1} = (b_{ij})$ inverse to $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{bmatrix}$ is equal to

A. $-\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. 2

E. 4

10. A is an $m \times n$ matrix and $\mathbf{b}$ is an $m \times 1$ vector. The equation $A\mathbf{x} = \mathbf{b}$ has **infinitely many** solutions. Consider the following statements:

(i) $m \leq n$

(ii) $n \leq m$

(iii) the rank of $A = n$

(iv) the rank of $A < n$

(v) $\det A = 0$

Which **must** be true?

A. only (i) and (v)

B. only (iv)

C. only (v)

D. only (iii) and (v)

E. None of the statements has to be true

(14) If

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

then the rank of A is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(18) Assume that the 4×4 matrix A is row equivalent to the matrix B, where

$$B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following statements is true?

(A) B is not the reduced row echelon form of A.

(B) $\det(A) \neq 0$.

(C) The null space of A has dimension 3.

(D) The column space of A has dimension 2.

(E) If $A\vec{x} = \vec{b}$ is consistent for some $\vec{b} \neq \vec{0}$, then it has infinitely many solutions.

11. If the dimension of the nullspace of a 5×6 matrix A is 4, what are the rank of A and the dimension of the column space of A?

A. rank(A)=3, dim[Col(A)]=3.
B. rank(A)=1, dim[Col(A)]=2.
C. rank(A)=2, dim[Col(A)]=1.
D. rank(A)=1, dim[Col(A)]=1.
E. rank(A)=2, dim[Col(A)]=2.

4. Which of the following statements about all 5×5 matrices is **true**?

A. $\det(A + B) = \det A + \det B$

B. $\det A^T = -\det(A)$.

C. $AB = 0$ implies $A = 0$ or $B = 0$.

D. If $\det A = 0$ then some two rows are proportional.

E. $\det(-A) = -\det(A)$.

7. What is the dimension of the space span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$?

A. 0

B. 1

C. 2

D. 3

E. 4

8. Let $A = \begin{pmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{pmatrix}$. Then the following is a basis of the nullspace of A.

A. $\{[2\ \ -4\ \ -8]^T, [-1\ 2\ 4]^T, [-2\ 4\ 8]^T\}$

B. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T\}$

C. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T, [3\ 2\ 2]^T\}$

D. $\{[1\ 0\ 1]^T\}$

E. $\{[0\ 2\ 1]^T\}$

9. Let A be an $m \times n$ matrix. Then the linear system $A\mathbf{x} = \mathbf{b}$ has a solution for any $m \times 1$ matrix $\mathbf{b}$ if and only if

A. $m = n$.

B. Nullity(A)$= m$.

C. Rank(A)+Nullity(A)$= n$.

D. $A = I$ the identity matrix.

E. Rank(A)$= m$.

7. Find the general solution to the differential equation

$$y^{(4)} - 8y'' + 16y = 0.$$

A. $y = c_1 e^{2x} + c_2 e^{-2x}$

B. $y = c_1 x e^{2x} + c_2 x e^{-2x}$

C. $y = c_1 e^{2x} + c_2 e^{-2x} + c_3 x e^{2x} + c_4 x e^{-2x}$

D. $y = c_1 x e^{2x} + c_2 x e^{-2x} + c_3 x^2 e^{2x} + c_4 x^2 e^{-2x}$

E. $y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

8. Let $y(x)$ satisfy

$$y'' + 9y' + 18y = 0, \quad y(0) = 0, \quad y'(0) = 3.$$

Then $y(\frac{1}{3}\log 3) = ?$

A. $y(\frac{1}{3}\log 3) = \frac{2}{9}$

B. $y(\frac{1}{3}\log 3) = \frac{2}{3}$

C. $y(\frac{1}{3}\log 3) = \frac{3}{4}$

D. $y(\frac{1}{3}\log 3) = \frac{1}{3}$

E. $y(\frac{1}{3}\log 3) = \frac{1}{4}$

16. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ -1 & -2 & -3 & -1 \end{bmatrix},$$

the dimension of $nullspace(A)$ is :

A. 4

B. 3

C. 2

D. 1

E. 0

3. Given that the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 3 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 1 & 2 & 2 & 3 & 12 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ is row-equivalent to the matrix $B = \begin{bmatrix} 1 & 2 & 2 & 3 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, which of the following are true?

I. A basis for Nul A contains exactly one vector.

II. A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ 1 \\ 0 \end{bmatrix} \right\}$.

III. A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ 12 \\ 1 \end{bmatrix} \right\}$.

IV. The column space of A is $\mathbb{R}^4$.

A. II and III only
B. I and IV only
C. I only
D. II only
E. III only

4. What is the dimension of the subspace of $\mathbb{R}^4$ consisting of all vectors in $\mathbb{R}^4$ whose first and third entries are equal?

A. 1

B. 2

C. 3

D. 4

E. 0

6. Which ones of the following statements are *always* true for an $m \times n$ matrix A?

I. The dimensions of the row space and the column space of A are always the same.

II. The sum of the dimensions of the rowspace of A and the null space of A equals the number of rows of A.

III. The dimension of the null space of A is the number of columns of A that are *not* pivot columns.

A. I only.

B. III only.

C. I and III.

D. I and II.

E. All of them.

5. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$, which of the following set is a basis of the column space of A?

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$

3. Let A be a 7×10 matrix of rank 5. Let x be the dimension of the null space of A, and y be the dimension of the null space of A^T, the transpose of A. What are x and y?

A. $x = 2$ and $y = 5$

B. $x = 2$ and $y = 4$

C. $x = 4$ and $y = 3$

D. $x = 5$ and $y = 4$

E. $x = 5$ and $y = 2$

1. (8 points) For what value of k is the vector $(-1, 2, 1, 1)$ in the span of $(1, 2, 1, -1)$ and $(2, 2, 1, k)$?

   A. 2

   B. 1

   C. 0

   D. -1

   E. -2

11. Find the solution to the initial value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

A. $y(t) = e^t + e^{-t}$
B. $y(t) = \cos t + \sin t$
C. $y(t) = e^t \cos t + e^t \sin t$
D. $y(t) = e^{-t} \cos t - e^{-t} \sin t$
E. $y(t) = e^{-t} \cos t + e^{-t} \sin t$

12. Consider the differential equation

$$y'' - 6y' + 9y = 0.$$

Which one of the following statements is true?

A. The equation has only one linearly independent solution e^{3t}
B. The equation has only one linearly independent solution te^{3t}
C. The equation has two linearly independent solutions e^{3t} and te^{3t}
D. The equation has two linearly independent solutions t and e^{3t}
E. The equation has two linearly independent solutions t^3 and e^{3t}

9. Let $y(x)$ satisfy the initial value problem

$$y'' + 2y' + y = 0,$$
$$y(0) = 3, \quad y'(0) = -2.$$

Then $y(1)$ (that is $y(x)$ evaluated at $x = 1$) is equal to

A. e^{-1}

B. $2e^{-1}$

C. $3e^{-1}$

D. $4e^{-1}$

E. $5e^{-1}$

10. Let $y(x)$ be the solution of the initial value problem

$$y'' - 4y = 4x^2,$$
$$y(0) = \frac{1}{2}, \quad y'(0) = 2.$$

Then $y(1)$ is equal to

A. $y(1) = e^2 - \dfrac{3}{2}$

B. $y(1) = e^2 + e^{-2} + \dfrac{2}{3}$

C. $y(1) = e^2 + e^{-2} - \dfrac{3}{2}$

D. $y(1) = e^{-2} - \dfrac{3}{2}$

E. $y(1) = e^2 - \dfrac{7}{2}$

7. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$, which of the following set is a basis of the null space of A?

A. $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

8. Let $y(t)$ be the solution to the initial value problem $y'' + 3y' - 4y = 6e^{2t}$, $y(0) = 2$, $y'(0) = 3$, find $y(1)$.

A. e
B. e^2
C. $e + e^2$
D. $e - e^2$
E. $2e - e^2$

10. Determine the general solution to the differential equation

$$y^{(5)} + 4y^{(4)} + 5y''' = 0\,.$$

A. $y = C_1 + C_2\,e^{-2x}\cos x + C_3\,e^{-2x}\sin x$

B. $y = C_1 + C_2\,x + C_3\,x^2 + C_4\,e^{-2x}\cos x + C_5\,e^{-2x}\sin x$

C. $y = C_1 e^x + C_2\,xe^x + C_3\,x^2 e^x + C_4\,e^{-2x}\cos x + C_5\,e^{-2x}\sin x$

D. $y = C_1 + C_2\,x + C_3\,x^2 + C_4\,e^{-x} + C_5\,e^{-5x}$

E. $y = C_1 + C_2\,x + C_3\,x^2 + C_4\,e^x\cos 2x + C_5\,e^x\sin 2x$

10. Find conditions for a, b and c such that the vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is in the column space of the matrix

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & 5 & 8 \\ 1 & -4 & -6 \end{bmatrix}$$

A. $a + b + c = 0$

B. $a - b - c = 0$

C. $2a - b + c = 0$

D. $3a + b - 2c = 0$

E. $8a + b + c = 0$